24.

$$\lim_{x\to 0^+} (\sin x)^x = y \qquad 0^0$$

$$\lim_{x\to 0^+} \ln(\sin x)^x = \ln y$$

$$\lim_{x\to 0^+} x\ln(\sin x) = \ln y$$

$$-\frac{\infty}{\infty} \qquad \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}} = \ln y$$

$$\lim_{x\to 0^+} \frac{\frac{1}{\sin x}\cdot\cos x}{-\frac{1}{x^2}} = \ln y$$

$$\frac{0}{0} \qquad \lim_{x\to 0^+} \frac{-x^2\cos x}{\sin x} = \ln y$$

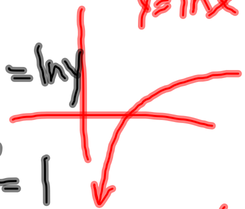
$$\lim_{x\to 0^+} \frac{-x^2(-\sin x) + -\cos x\cdot 2x}{\cos x}$$

$$= \frac{0}{1} = 0 = \ln y$$

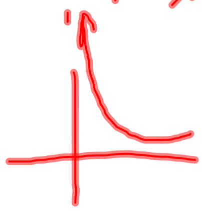
$$y = e^0 = 1$$

$$y = 1$$

y = ln x



$y = \frac{1}{x}$



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38.

$$\lim_{x\to 0^+} \ln x - \ln(\sin x)$$

$$\lim_{x\to 0^+} \ln\left(\frac{x}{\sin x}\right) = 0 \qquad \frac{x}{\sin x} \to 1$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

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8.3 relative rates of growth

(which function grows faster? or slower)

if $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} = \infty$ then $f(x)$ is faster

$\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} = 0$ then $g(x)$ is faster

$\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} = L$ $0 < L < \infty$, f & g ~~are same~~ grow at the same rate

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compare $y = x^2$ with $y = e^x$

$\frac{\infty}{\infty}$ $\lim\limits_{x\to\infty} \frac{e^x}{x^2} =$

$\lim\limits_{x\to\infty} \frac{e^x}{2x}$

$\lim\limits_{x\to\infty} \frac{e^x}{2} = \infty$

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$y = x \qquad y = \ln x$

$\dfrac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{x}{\ln x}$$

$$\lim_{x\to\infty} \frac{1}{\frac{1}{x}} = \infty$$

x grows faster

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transitive property

if $a=b$ & $b=c$ then $a=c$

$f(x) = \sqrt{x^2+5}$

$g(x) = (2\sqrt{x}-1)^2$

compare with $y = x$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{x} = \lim_{x\to\infty} \frac{\sqrt{x^2+5}}{\sqrt{x^2}} = \lim_{x\to\infty} \sqrt{\frac{x^2+5}{x^2}} = 1$$

same

$$\lim_{x\to\infty} \frac{(2\sqrt{x}-1)^2}{x} = \lim_{x\to\infty} \frac{(2\sqrt{x}-1)^2}{(\sqrt{x})^2} = \lim_{x\to\infty} \left(\frac{2\sqrt{x}-1}{\sqrt{x}}\right)^2 = 4$$

same

f(x) and g(x) grow at the same rate

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